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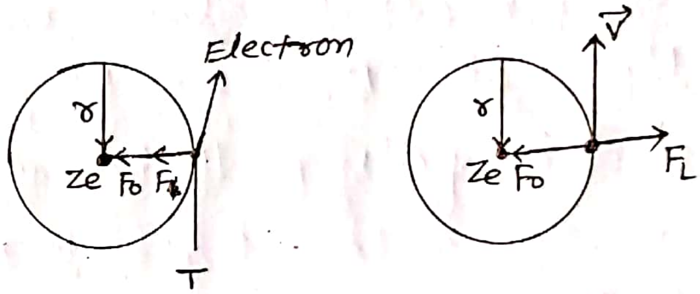
B.Sc (Part-II) Hons

Langevin's theory of Diamagnetism:-

Langevin's theory of Diamagnetism:— Langevin gave a mathematical theory of diamagnetism based on the electron theory of matter.



According to this theory, matter is atomic (or, molecular) in structure consisting of an assembly of electrons revolving in closed orbits around positive nuclei. An electron moving in an orbit is ~~equal~~ equivalent to an electric ~~energy~~ current given by.

$$I = \frac{e}{T}$$

where e is the electronic charge and T is the period of revolution of electron.

If ω_0 be the angular velocity, then

$$T = \frac{2\pi}{\omega_0}$$

$$\therefore I = \frac{e\omega_0}{2\pi}$$

Let us consider an electron of mass m and charge e rotating about the nucleus of charge (ze) in a circular orbit of radius r.

$\therefore$ Electrostatic force between electron and nucleus $= \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze \cdot e}{r^2}$

This is balanced by the centrifugal force provided by the circular motion of electron which is

$$F_0 = \frac{mv^2}{r}$$

$$F_0 = m\omega_0^2 r$$

$$\therefore m\omega_0^2 r = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r^2}$$

$$\therefore \omega_0^2 = \frac{Ze^2}{4\pi\epsilon_0 \cdot mr^3}$$

$$\therefore \omega_0 = \sqrt{Ze^2/4\pi\epsilon_0 \cdot mr^3} \quad \text{——— (1)}$$

Magnetic momentum of electron is

$$\vec{m} = \text{Current} \times \text{area}$$

$$= \frac{e\omega_0}{2\pi} \times \pi r^2$$

$$\vec{m} = \frac{e}{2}\omega_0 r^2 \quad \text{——— (2)}$$

Let us suppose that a magnetic field induction $\vec{B}$ is now applied where $\vec{B}$ is normal to and into the page as shown in the figure. An additional Lorentz force $\vec{F_L}$ acts as the electron where,

$$\vec{F_L} = -e(\vec{V} \times \vec{B}) = -eBr\omega$$

∴ The condition for stable motion is

$$mr\omega^2 = \frac{Ze^2}{4\pi\epsilon_0 r^2} - eBr\omega$$

$$\text{or, } \omega^2 = \frac{Ze^2}{4\pi\epsilon_0 m r^3} - \frac{eB}{m}\cdot\omega$$

$$\text{or, } \omega^2 + \frac{eB}{m}\omega - \frac{Ze^2}{4\pi\epsilon_0 \cdot m r^3} = 0$$

which is a quadratic equation in ω

$$\therefore \omega = \frac{-\frac{eB}{m} \pm \sqrt{\left(\frac{eB}{m}\right)^2 + 4\left(\frac{Ze^2}{4\pi\epsilon_0 m r^3}\right)}}{2}$$

$$= \frac{-\frac{eB}{m} \pm \sqrt{\left(\frac{eB}{m}\right)^2 + 4\omega_0^2}}{2}$$

$$= \pm\sqrt{\omega_0^2 + \left(\frac{eB}{2m}\right)^2} - \frac{eB}{2m}$$

$\because \frac{eB}{2m} << \omega_0$, $\left(\frac{eB}{2m}\right)^2$ can be neglected

$$\therefore \omega = \pm\left(\omega_0 - \frac{eB}{2m}\right) \quad \text{———} \quad (4)$$

Thus the angular frequency is now different from ω_0. The flux density $\vec{B}$ sets up a precessional motion of electronic orbit with angular velocity $-(e/2m)B$. This is called Larmor theorem.

$\therefore$ change in frequency of revalution of electron.

$$d\nu = -\frac{eB}{4\pi m}$$

$\therefore$ Corresponding change in magnetic moment

$$\Delta m = \text{Current} \times \text{area}$$

$$= e \times \left(\frac{-eB}{4\pi m}\right) \times \pi r^2$$

$$\Delta m = -\frac{Be^2 r^2}{4m} \quad \text{———} \quad (5)$$

$\therefore$ Induced moment per atom is

$$(\Delta m)_{atom} = -\frac{Be^2 \cdot \Sigma r^2}{4m}$$

Let N be the number of atoms per unit valume. Then the magnetisation

$$M = -\frac{NBe^2 \Sigma r^2}{4m} \quad \text{———} \quad (6)$$

Since all the electron orbits are not oriented to the magnetic field, hence r^2 in equation (6) should be replaced by the average of the square of the projection of orbit radii for various electrons in a plane perpendicular to B. Hence r^2 should be replaced by $\frac{2}{3} r^2$

$$\therefore M = -\frac{NBe^2 \Sigma r^2}{6m}$$

$$\therefore \chi = \frac{M}{H}$$

$$= -\frac{NBe^2 \Sigma r^2}{6mH}$$

$$= -\frac{N\mu_0 He^2 \Sigma r^2}{6mH}$$

$$\therefore \chi = -\frac{\mu_0 Ne^2 \Sigma r^2}{6m}$$

$$\chi = -\frac{\mu_0 e^2}{6m} N \Sigma r^2$$

$\therefore \chi$ is independent of the field strength and temperature.